Maheshtala College
CU UG Intermediate Examination–2020
B.Sc. Mathematics (Honours)
SEM-4(CC–X)

Time : 2PM-3PM **Full Marks : 50**
DATE : 05.12.2020

Group-A
Answer any <u>TWO</u> questions : [$2 \times 16 = 32$]

1. A particle of mass m moves under a central attractive force $m\mu(5r^{-3} + 8c^2r^{-5})$ and is projected from an apse at a distance c with a velocity $\frac{3\sqrt{\mu}}{c}$, Prove that the orbit is $r = c\cos\frac{2}{3}\theta$, show further that it will arrive at the origin after a time $\frac{\pi a^2}{8\sqrt{\mu}}$. 16

2. A particle is projected upwards with a velocity U in a medium whose resistance varies as the squire of the velocity, will return to the point of projection with a velocity $v_1 = \frac{UV}{\sqrt{U^2+V^2}}$ after a time $\frac{V}{g}(tan^{-1}\frac{U}{V} + tan^{-1}\frac{v_1}{V})$, where V is the terminal velocity. 16

3. A heavy uniform rod AB of length $2a$ rest with its end in contact with smooth inclined plane of inclination α and β to the horizon. If θ be the inclination of the rod of the horizon, then show that by principle of virtual work $\tan\theta = \frac{1}{2}(\cot\alpha - \cot\beta)$. 16

4. A particle of mass m moves under a central force $m\mu\{3au^4 - 2(a^2 - b^2)u^5\}$, $a > b$. It is projected from an apse at a distance $(a + b)$ with a velocity $\frac{a+b}{\sqrt{\mu}}$. Show that the path is $r = a + b\cos\theta$. 16

Group-B
Answer any <u>ONE</u> question : [$1 \times 8 = 8$]

5. A particle describes the path $r = a\tan\theta$ under a force to the origin. Find the acceleration and velocity in terms of r. 8

6. If the central acceleration be $\frac{\mu}{r^5}$, then prove that the velocities v_1 and v_2 at the two apsidal distances satisfy the relation $v_1^2 + v_2^2 = \frac{2h^4}{\mu}$. 8

Group-C
Answer any <u>ONE</u> question : [$1 \times 10 = 10$]

7. A force parallel to the axis of z acts at the point $(a, 0, 0)$ and an equal force perpendicular to the axis of z acts at the point $(-a, 0, 0)$.
Show that the central axis of the system lies on the surface $z^2(x^2 + y^2) = (x^2 + y^2 - ax)$. 10

8. A rhombus ABCD of four equal uniform rods freely joined together and suspended from the point A, it is kept in position by a light rod joining the mid points of BC and CD, if T be the thrust in the rod and W be the weight of the rhombus, prove that $T = W\tan\frac{1}{2}A$. 10